

# Proposal of an Alternative Model for Speed-Flow Relationship in Two-Lane Highways

Manuel G. Romana<sup>a</sup>, Miguel Núñez<sup>b</sup> \*

<sup>a</sup>Technical University of Madrid, ETS Caminos, Profesor Aranguren s/n 28040 Madrid, Spain

<sup>b</sup> Comunidad de Madrid, Sor angela de la Cruz 60,28036 Madrid, Spain

## Abstract

A research has been carried out in two-lane highways in the Madrid Region to propose an alternative model for the speed-flow relationship using regular loop data. The model is different in shape and, in some cases, slopes with respect to the contents of Highway Capacity Manual (HCM). A model is proposed for a mountainous area road, something for which the HCM does not provide explicitly a solution. The problem of a mountain road with high flows to access a popular recreational area is discussed, and some solutions are proposed. Up to 7 one-way sections of two-lane highways have been selected, aiming at covering a significant number of different characteristics, to verify the proposed method the different classes of highways on which the Manual classifies them. In order to enunciate the model and to verify the basic variables of these types of roads a high number of data have been used. The counts were collected in the same way that the Madrid Region Highway Agency performs their counts. A total of 1.471 hours have been collected, in periods of 5 minutes. The models have been verified by means of specific statistical test (R<sup>2</sup>, T-Student, Durbin-Watson, ANOVA, etc.) and with the diagnostics of the contrast of assumptions (normality, linearity, homoscedasticity and independence). The model proposed for this type of highways with base conditions, can explain the different behaviors as traffic volumes increase, and follows a polynomial multiple regression model of order 3, S shaped. As secondary results of this research, the levels of service and the capacities of this road have been measured with the 2000 HCM methodology, and the results discussed.

*Keywords:* Two-lane highways, Speed-flow relationship, Free-flow speed

---

## 1. Introduction

The fourth edition of Highway Capacity Manual, published in 2000 by TRB (National Research Council, USA, 2000) describes the speed-flow relations as parallel lines of constant slope equal to -0,0125 km/h for each vehicle that travels on the road, in both directions. There have been no new developments in this field in HCM 2010 (Transportation Research Board, 2010). The simplicity of this model, which does not incorporate the peculiarities detected in the appraisals made in Madrid, and who is unable to explain the differences between traffic states, makes

it very difficult to apply the Capacity Manual 2000 or 2010 in the conventional roads in Madrid Region. Thus, the goal of this research is establishing a model that can be used in conjunction with the HCM methodology to determine level of service in these facilities

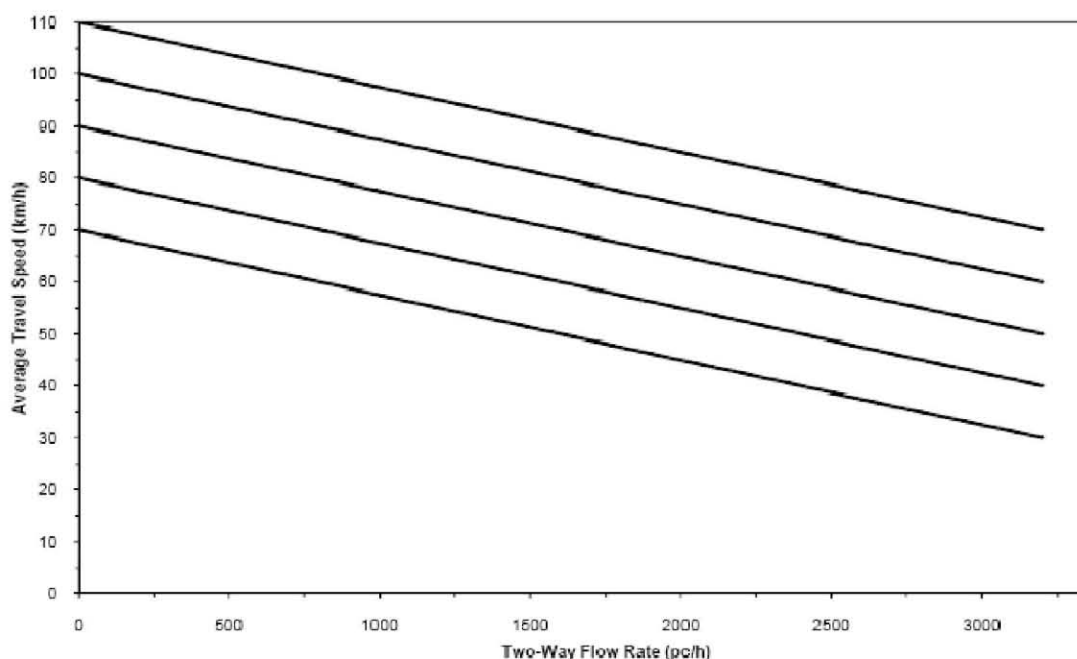


Figure 1. Flow-speed functions for two-lane roads with base conditions. Source: Transportation Research Board (2000).

## 2. Sections Considered

The research framework has focused on the scope of Madrid Region. The Highway Agency of Madrid Region has a two-lane conventional roads of various kinds (urban, interurban and rural areas), in different terrain (level, rolling and mountainous) and with very different ADTs, etc. The research has been carried out in highways with very different characteristics, aiming to cover as many two-lane highway types as possible. The data were collected at counting stations in the network.

Table 1. Sections considered for data collection and location of permanent stations

Highway	Section	Station	ADT (2008)	Terrain
M-509	M-851 – Villanueva del Pardillo	3+430	26.003 v/d	Level-Rolling
M-305	A-4 – Aranjuez	1+920	25.124 v/d	Level
M-601	M-607 – Puerto de Navacerrada	14+240	4.909 v/d	Mountainous

## 3. Collected Data

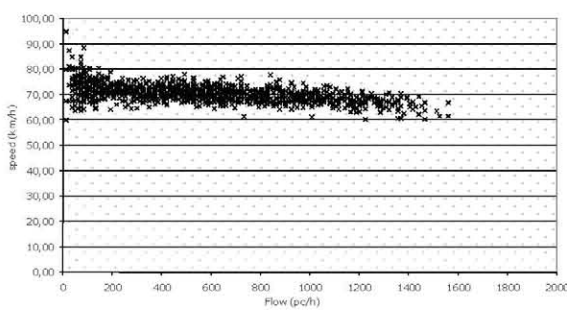
In each of the stations five minutes counts were acquired. Considering the whole of all selected stations 14.177 periods of five minutes of data were measured.

Table 2. Collected data

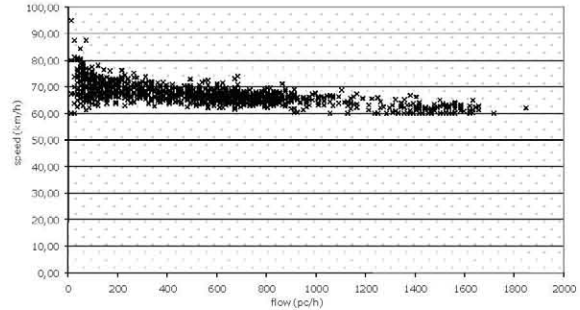
Highway	Station	Direction	Date 1	Date 2	Data
M-509	Pk 3+430	Direction 1	06/02/08 (09:30)	11/02/08 (10:20)	1.451
M-509	Pk 3+430	Direction 2	06/02/08 (09:30)	11/02/08 (10:20)	1.451
M-305	Pk 1+920	Direction 1	06/02/08 (09:25)	11/02/08 (10:15)	1.451
M-601	Pk 3+430	Direction 1	13/12/07 (10:15)	30/12/07 (11:35)	4.912
M-601	Pk 3+430	Direction 2	13/12/07 (10:15)	30/12/07 (11:35)	4.912

In addition, a careful inventory of the geometric characteristics of the selected road sections was performed, something that is needed for the interpretation of the collected data. All data used in this research are field data, without use of simulation. The first conclusions when the data were plotted are the following:

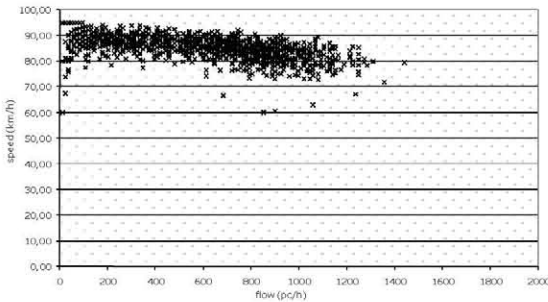
- The relationships differed in shape and maximum observed flows.
- The shapes did not look linear.
- There was considerable noise.
- It was not possible to isolate the effect of heavy vehicles in these roads.



(a)



(b)



(c)

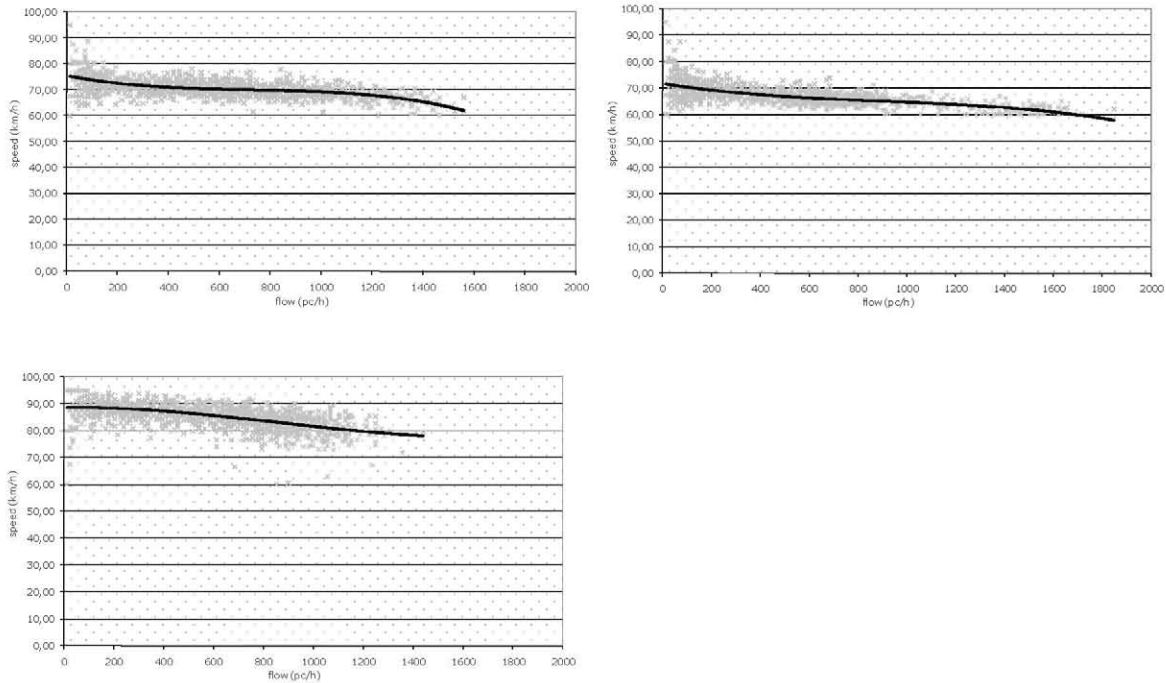
Figures 2a to 2c – Speed-flow data collected at different stages.

#### 4. 3<sup>rd</sup> Degree Polynomial Multiple Regression Model with All Data Available

It has been considered and that the curve which is the best fit the configuration of the data for different road sections is described by a polynomial multiple regression model. This assumption was verified using statistical analysis tools. An approach to vary the degree of the polynomial function was tested using strength measuring statistical parameters (coefficient of determination, correlation, T-Student, Durbin-Watson, etc.), and it was concluded that a 3<sup>rd</sup> degree polynomial model was the best approach to describe the relationships (Blank, 1980, and . Fisher and Drake, 1953).

The average speed-flow curves for the road sections analyzed considering the data of all vehicles (light and heavy) are projected with a polynomial trend of 3<sup>rd</sup> order in the next figure. As it can be seen, the 3<sup>rd</sup> degree polynomial function is able to reflect the general shape of the curves.

This shape has not been tried by other authors, that have favored straight or second degree models (Luttinen et al, 2005), Yet they are clearly perceived in some reported data (Luttinen, 2008)



Figures 3a to 3c – 3rd degree polynomial curve for the relationships average speed - flow.

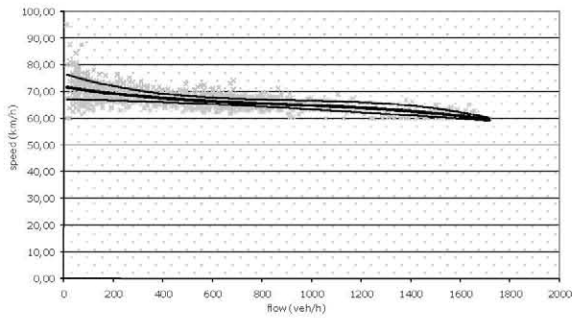
The shape of the curve (S-shape) has three different regions. The first part of the function has a concave shape, and represents the free flow section, in which vehicles can pass, and, therefore, slower vehicles do not condition the overall performance of the facility. Next, the second part of the function, intermediate movement, has a convex shape, which starts from the flows when traffic conditions cause friction between the users. Finally, the last part of the curve includes the flows from which the freedom of movement is small, and thus a vehicle is restricted to forced circulation. Approximation curves for sections that are far from base conditions will soften in the initial concave shape, almost eliminating the convex-shaped region.

## 5. Consistency of the 3rd Degree Polynomial Multiple Regression Model

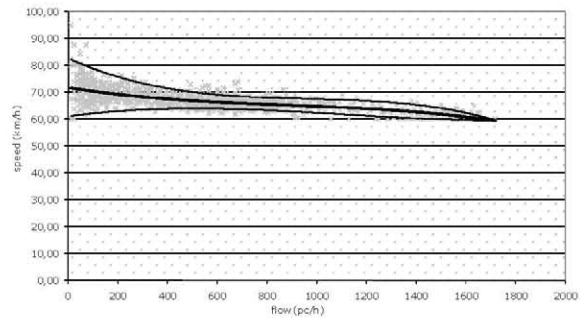
The consistency of these functions has been verified by using statistical methods. The variance of all data collected with a flow  $i$  is determined for each of the flow values ( $I_i$ ), and the approximation function to the current-speed curve with wholesale and retail function of the value 95% compliance with the variance in each  $I_i$  was represented. Thus for every  $I_i$ ,

$$s_i^2 = \sum (u_i - \bar{u})^2 / (n - k - 1) = \sum e_i^2 / (n - k - 1) \quad (1)$$

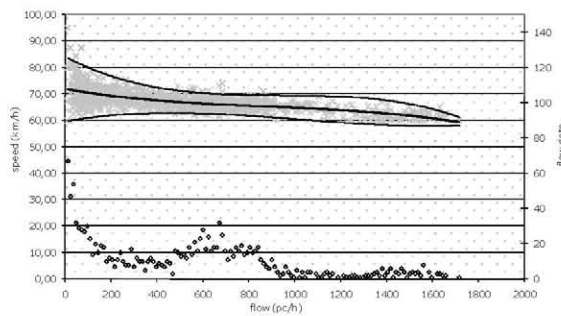
where the variance,  $s_i$ , is equal to the sum of the squares of the mistakes in every data collection,  $\sum e_i^2$ , divided by the degrees of freedom;  $n$  is the number of observations in each measure  $i$ , and  $k$  the number of independent variables, which in this case would be 3.



(a)



(b)



(c)

Figures 4a to 4c - Consistency of the speed-flow curve mean difference speed by means of average difference of speeds, maximum absolute difference and values with 95 % of the variance.

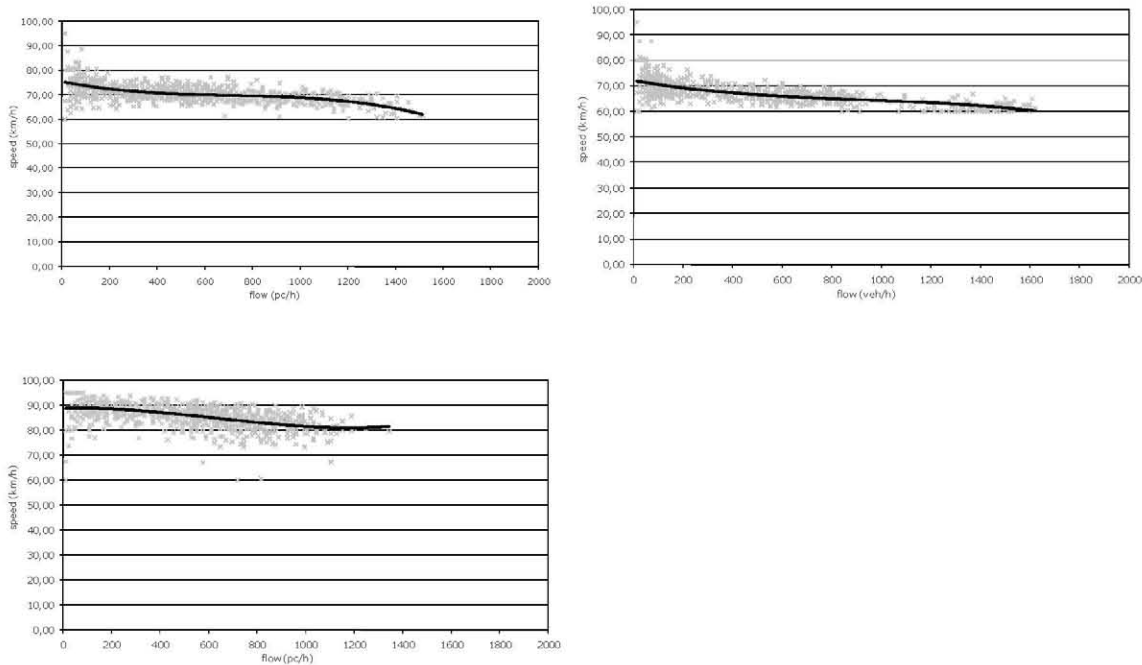
## 6. Approximation of the Speed-Flow Function in Two-Lane Highways with Base Conditions

If it is not possible to obtain all the necessary information to represent the average speed-flow relationship particularized for a specific road section we must turn to a regression model. Base conditions for this model should be adjusted according to these conditions (lanes 3,5 m wide, no obstacles on the roadsides, shoulder characteristics, free overtaking, and all vehicles being light vehicles).

Therefore, the regressions themselves would only be valid in the highways where the measurements were taken. To improve the quality of the regressions and, more important, to establish a model that could be used in other highways, two modifications were adopted

- Noise caused by heavy vehicles. To study the relationship in ideal conditions, it is necessary to eliminate from the data the points that have such influence. In order to do this, all periods where the difference in average speeds between light and heavy vehicles were taken out of the sample. This is adequate particularly when considering speed and not queue structure (PTSF in the HCM).
- Eliminate the noise caused by speed dispersion with low flows. After some consideration, all flows under 200 veh/h were taken out of the samples. Other authors limit their laws to start at a certain range of flows, and propose no relationship for lower flows. In this case, the significance analysis proved that when considering only flows greater than 200 veh/h, the curves can be approximated with very similar results with a 2<sup>nd</sup> or 3<sup>rd</sup> order polynomial. This is consistent with the hypothesis, since by doing this the first region was obliterated. It was also found that the difference between the 2<sup>nd</sup> and 3<sup>rd</sup> order polynomials was negligible. Therefore, two options emerged: using, for simplicity, a 2<sup>nd</sup> order function with flows greater than 200 veh/h, or using a 3<sup>rd</sup> degree function when considering the entire range of flows for a stretch of road with base conditions.

After these modifications, the speed flow curves were obtained (figure 5). This laws should reflecto base conditions with no influence of heavy vehicles.



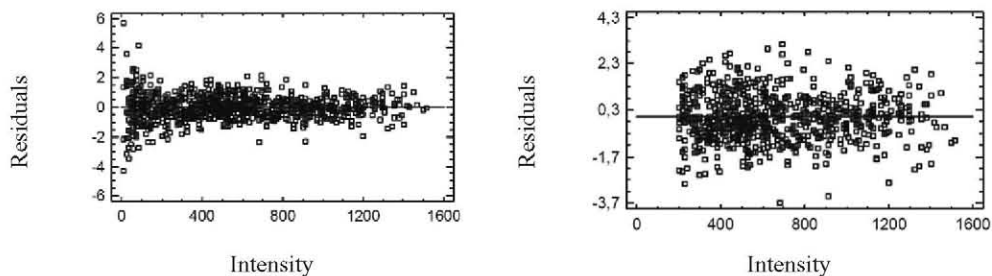
Figures 6a to 6c - shape with base conditions.

## 7. Hypothesis Testing Of 3<sup>rd</sup> Degree Regression Model For Speed-Flow Relationships in Base Conditions

In order to confirm that the 3rd degree polynomial multiple regression model is capable of reproducing a statistically acceptable model for the average speed-flow curves in rural highways in Madrid Region in base conditions, some hypotheses tests were carried out. In particular, the aim was to establish the regression assumptions of normality, linearity, homoscedasticity and independence. It has used the statistical software Statgraphics Plus 5.1.

It was noted that if low flows were included, particularly for values below 200 veh/h, the 3rd degree polynomial model tends to heteroscedasticity, explained by the free circulation region, where the divergence in the velocity pattern is due the driver variation, because users can move at the speed of their own choosing, without other conditions that cause a more uniform channeling as happens with higher flows.

On the other hand, it has been shown statistically that the relationships for flows above 200 veh/h is reproduced by a 2nd degree polynomial multiple regression model, verifying compliance with the hypothesis of homoscedasticity, linearity, normality and independence. These models do not show signs of heteroscedasticity.



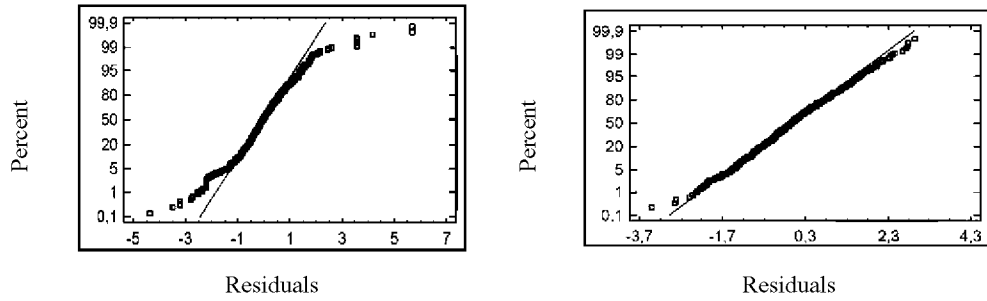


Figure 6 – Road M-509. D1, residuals and normality of residuals with all flows (6a and 6b) and with flows over 200veh/h (6c and 6d).

## 8. Results

A 3rd degree model, statistically valid for all regions, and with all regressions assumptions tested for flows above 200 veh/h is proposed. In this model there are three regions: the first one in which the dispersion of data is larger (low flows), the second one with a stable speed (intermediate flows) and the third one where the circulation is highly conditioned, and there is no overtaking. This model has been tested for homoskedasticity, linearity, normality and independence, and the usual regression tests. The model is, thus

$$y = A x^3 + B x^2 + C x + D \quad (2)$$

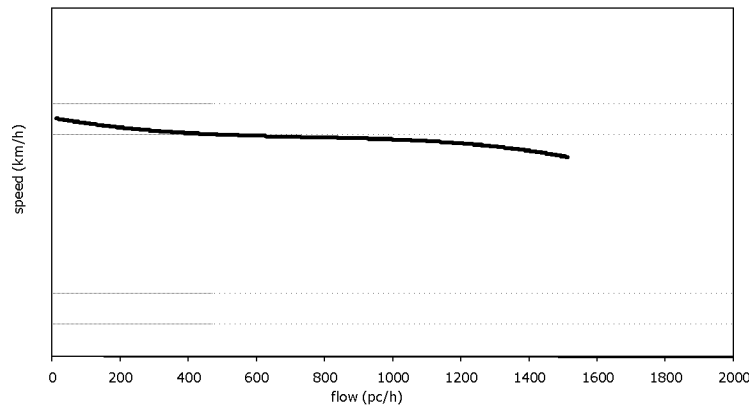


Figure 7 –Proposed shape for the average speed-flow curves with base conditions in two-lane roads.

Data with flows under 200 veh/h introduce some heteroscedasticity in the model, but the inclusion of these data in the curve does not introduce any error in the points of the regression for flows above 200 veh/h (i.e., the 2<sup>nd</sup> and 3<sup>rd</sup> degree polynomials coincide (less than 5% difference) on the region beyond 200 veh/h). If these low flows are discarded, the model could be approached with the same quality with a 2nd degree polynomial. For simplicity, this should be preferable.

In this low-flow region, the vehicles circulate at their drivers' desired speeds, so there is a greater dispersion of data. Also, since passing is much easier, slow-moving vehicles will have a greater effect on faster vehicles as flows increase. To include this first region, and this convex shape, a 3rd degree model is needed. The term  $Ax^3$  is completely necessary in order to adequately describe this region. When overtaking is not allowed, or is difficult, as in rolling or mountain environments, the convex region tends to disappear. Also, if the sections are long (actually, as the length of the section grows longer) the probability of a fast vehicle reaching a slow vehicle increases, the need for passing increases, and this region will shrink, possibly even below 200 veh/h, This is likely to be the reason that some authors (Werner and Brilon, 2005)

Regarding the second region, termed intermediate movement (for  $I > 200$  veh/h until the overtaking is almost impossible), the best fit of the data is achieved by a concave shape. This concavity begins to disappear from the

moment where there is considerable friction between vehicles, when most vehicles are following other somewhat slower vehicles. Until that moment, some fast vehicles are still travelling unimpeded, but as flows increase they will reach sooner a slower vehicles, thus lowering the overall average speed more markedly than a linear function (concavity appears). It represents the transition area between the free flow, the 1st region, and the forced flow, the 3rd region.

The 3rd, and last part of the curve, represents largely a forced flow of vehicles, when most fast vehicles travel behind slower vehicles. The density of relatively slower vehicles is high, so there is a low probability of a fast vehicle covering large subsections of the studied section at desired speeds. From this point on, passing is almost impossible, and freedom of movement is practically null; therefore, platoons constitute overwhelmingly the majority of the flow. In this region there are virtually no safe overtakings, and practically do not happen.

Consequently, in roads with characteristics close to ideal the shape of the curve follows a 3rd degree model, which is capable of reproducing the greater dispersion that occurs with  $I < 200$  veh/h, while in other sections (with a high percentage of prohibition of overtaking in mountain environments, etc..) is preferable a 2nd degree model, which contains graphical representation 2nd and 3rd regions.

## 9. Conclusions

The final conclusions of this research for developing a proposal of an alternative model for speed-flow relationship in two-lane highways are the following:

1. It is not recommended to apply directly the methodology proposed by the Highway Capacity Manual 2000 for determining the flow-speed characteristics of rural two-lane highways (linear regression model) which does not reproduce correctly the modes of movement on conventional roads in Madrid Region.
2. It is proposed to use a S-shaped 3rd degree polynomial multiple regression model to represent the speed-flow relationship on roads. This relationships should be developed specifically for each road sections where it is possible to obtain real data. These representations include the need to interpret the curve, slope and boundaries of each of their areas according to the characteristics of the stretch, both alignment and cross.section characteristics, such as the environment.
3. It is proposed that for those roads or road sections where it is not possible to obtain real data, the curves proposed are used

## 10. References

- [1] BLANK, L. (1980): Statistical Procedures for Engineering, Management and Science. McGraw-Hill, New York.
- [2] BRILON, W. and WEISER, F. (2005) Two-Lane Rural Highways – the German Experience. Paper 06 -1625, 85th TRB Annual Meeting, Washington, 2005
- [3] FISHER, R.A. y YATES, F. (1953): Statistical tables. Fourth edition. Oliver and Boyd. Edinburg Tweeddale Court, London. England.
- [4] LUTTINEN, R.T. (2008). Some effects in two-lane highways. Presentation at the two-lane subcommittee meeting, HCQS Committee, Transportation Research Board, January 2008, Washington
- [5] LUTTINEN, R.T., DIXON, M. and WASHBURN, S. (2005). Two-Lane Highway Analysis in HCM2000. Draft white paper. Unpublished.
- [6] MAY, A. D. (1990): Traffic Flow Fundamentals. Prentice Hall, Englewood Cliffs, New Jersey.
- [7] TRANSPORTATION RESEARCH BOARD (2010): Chapter 15: Two-lane highways. Highway Capacity Manual 2010. TRB, National Research Council.
- [8] TRANSPORTATION RESEARCH BOARD (2000): Highway Capacity Manual 2000. TRB, National Research Council, Washington, D.C.